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Paulo Júlio
Susana Peralta

Av. da República nº. 79 1050 – 243 Lisboa
Telf: (351) 217998150
Fax: (351) 217998154
Web Site: www.gee.min-economia.pt

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Paulo Júlio

Gabinete de Estratégia e Estudos, Ministry of Economy, Innovation and Development.

Susana Peralta

Faculdade de Economia, Universidade Nova de Lisboa, CORE-UCL and CEPR.

Classificação JEL codes: D70, H11, H41, H70

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September 30, 2010

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*Gabinete de Estratégia e Estudos, Ministry of Economy, Innovation and Development. Email: paulo.julio@gee.min-economia.pt.

†Corresponding author. Faculdade de Economia, Universidade Nova de Lisboa, CORE-UCL and CEPR. Email: peralta@fe.unl.pt.

1 Introduction

Interjurisdictional spillovers have been around in the debate about the relative merits of centralization and decentralization since Oates (1972). A textbook example of interjurisdictional spillovers consists in residents of one locality enjoying local amenities such as parks in a neighbor jurisdiction (Bloch and Zenginobuz, 2007). It is clear that this possibility depends on the travel distance between the two jurisdictions, which depends on the level of infrastructures, itself a public good. This particular type of public good varies widely across countries, as shown recently by Calderón and Chong (2004), who report a "quantity of infrastructure" index which ranges from 0.33 to around 500 in telecommunications, and from 0 to 3.81 in roads. Even when restricting the sample to developing countries alone, the authors report a range of 0.33 to 210.97 and 3.17, for the same types of infrastructures. Slightly lower, but still considerable ranges of variation are reported for the "quality of infrastructure" index. The relationship between quality of infrastructure and travel costs has been established empirically by, *inter alia*, Limão and Venables (2001) and Canning (1998). Another example is a multi-lingual country where the citizens may enjoy cultural goods provided in the other language. Their ability to do so depends heavily on investment in foreign language training as a part of education policy. In Spain, for instance, each region shares its local language with a common one (the *castellano*) whereas Belgium and Switzerland are examples of countries where local languages do not overlap with a common one. While in Switzerland there is an obligation to learn at least another official language, this is not the case in Belgium. Language barriers have been shown to create labor market segmentation (Cattaneo and Winkelmann, 2005). The literature on psycholinguistics has also long recognized the high cost imposed by modern education systems in natives of minoritarian languages. As put by Mohanty (2009), *"The relationship between language and power makes it a world of unequal languages. Languages of the marginalized people are treated with discrimination at all levels of the society (...) Languages of the disadvantaged entail disadvantages in a society that deprives them of their legitimate place in a multilingual structure."*

The analysis of issues related to fiscal decentralization dates back to Tiebout (1956) and Williams (1966), although it was not until the influential *decentralization theorem* in Oates (1972) that the trade-off between centralization and decentralization has been emphasized. The theorem asserts that centralization outperforms decentralization when the costs imposed by uniform provision of local public goods are outweighed by the benefits from internalizing spillovers.[1] Oates' assumption of policy uniformity has been challenged often by the literature due to its *ad hoc* nature. Recent contributions put politics and institutions at the heart of the debate, taking a political economy viewpoint.[2] In Besley and Coate (2003), centralized decision is undertaken by an assembly of locally elected representatives. They analyze both a cooperative and non-cooperative legislature. Under the latter, the minimum winning coalition is the representative of one of the jurisdictions, who then tilts public good provision in favor of her own jurisdiction. In addition, both constituents have equal *a priori* probabilities of holding office, hence there is policy uncertainty involved. A cooperative legislature, in turn, delegates decision making to public good lovers, thus leading to over-provision in the centralized regime. Some recent papers, like Dur and Roelfsema (2005) and Schnellenbach et al. (2010), use Besley and Coate's approach to study cost sharing and the decision to centralize in direct and representative democracies, respectively. A recent paper by Koethenbuerger (2008) revisits the standard Oates's argument using benevolent governments and iso-elastic preferences for public consumption. Interestingly, he shows that the relative advantage of centralization need not be monotonic in the degree of spillovers. However, this argument never changes the bottom line of the decentralization theorem, *i.e.*, there is always a threshold level of spillovers above which centralization is the dominant regime.

This paper takes a fresh look at the trade-off between centralization and decentralization in a model where the degree of spillovers depends on a distance-decreasing discrete public project, which we call the *bridge* (one may see different languages as cultural distance, see Ginsburgh et al., 2005). We argue that in a decentralized system

[1]This appealing argument has given rise to an extensive literature incorporating interjurisdictional spillovers in different setups (e.g. Koppel, 2005; Rubinchik-Pessach, 2005; Akai and Mikami, 2006; Bloch and Zenginobuz, 2007).

[2]For a careful survey on fiscal decentralization under a political economy perspective, see Lockwood (2006). Examples of mechanisms that may tilt the debate in favor of one of the regimes include whether one of the government levels is more prone to capture by special interests (see Redoano, 2010, which relates lobbying to the degree of heterogeneity in the population, Bardhan and Mokherjee, 2000, for a political competition setup, and Besharov, 2002 for a menu-auction one), and political agency (Hindriks and Lockwood, 2004; Belleflamme and Hindriks, 2005). A recent contribution by Tommasi and Weinschelbaum (2007) models the trade-off between spillover internalization and the improved accountability of decentralization, in a political agency model where citizens sign up contracts with the political representatives under different assumptions about effort observability and principals' coordination.

local governments decide on the level of local public goods, while the central government decides upon the bridge construction. A centralized system, in turn, allocates all the decision power to the same government level. To the best of our knowledge, the only example in the literature which opens the black-box of inter-jurisdictional spillovers is Strumpf (2002), who proposes a model of learning, whereby local governments learn from each other's policy experiments. The idea that spillovers depend on strategic decision making has been studied in the context of firms. Two recent examples include Piga and Poyago-Theotokyb (2005), who relate the level of spillovers to firms' location choices, and Gersbach and Schmutzler (2003), where the spillover depends from successfully biding the competitor's employee. Amir et al. (2003) suggest that the degree of endogenous R&D spillovers may result from firms' location, hiring away each other's scientists, camouflaging their products and processes more or less intensively, agreeing to choose more or less differentiated products, or more or less related R&D approaches or paths. We borrow from the idea that a lower distance implies a greater degree of spillovers. Given that local jurisdictions are geographically immobile, distance is decreased through a discrete public project provided by the central government.

We model an economy composed of two heterogeneous jurisdictions of unequal size, where the median voters have different tastes for the local public good. In each jurisdiction there is a local public good, which benefits local and, possibly, neighbor residents, in case the bridge is built. Under decentralization, the provision of local public goods is set by the local government, while, in a centralized regime, this task is left to the central authority. The bridge, in turn, is always decided by the central government. Not surprisingly, the decision to build the bridge is always characterized by a cut-off cost level. When the bridge is too costly, it does not pay to build it. Interestingly, while with benevolent governments *à la* Oates the cut-off is always higher in the centralized regime, *i.e.*, the bridge is built more often under centralization, this need not be the case when decisions are taken by majority elected representatives.

We compare total welfare levels across the two regimes. The endogenous spillovers framework generates three different ranges in the space of the bridge cost (which depend on the decision making rule and on which district hosts the majority of the citizens). When the cost is low, the bridge is built under both regimes. When the cost is intermediate, one regime builds the bridge while the other does not and, for sufficiently high costs, the bridge is not built under both regimes. We show that in the first (low cost) range the usual insight that centralization dominates when the spillovers (which we refer to as the bridge's *quality*) are high drives the results. In the third (high cost) range decentralization always dominates. Actually, decentralization always dominates when centralization does not build the bridge, irrespective of whether the building costs are high (no bridge in both regimes) or intermediate (in the cases where the bridge is built under decentralization but not under centralization). This stems from the fact that, in the absence of the bridge, there are no gains from centralized decision making. This corresponds to the worst case scenario in the political economy approach. This is because the local jurisdiction holding the central government also opts not to provide any local public good in the neighbor jurisdiction, since it cannot enjoy it without a bridge.

Finally, it may happen that for an intermediate cost range, centralization builds the bridge, while decentralization does not (under both the benevolent government and the median voter setups). Then it may still happen that decentralization is the preferred regime. This is a natural consequence of the non-optimality of the decision on the bridge construction, which arises even with benevolent governments, since it is decided taking into account sub-optimal local public good levels.

We also show that a centralized regime with separation of powers, in which one local representative decides both local public good levels, and the other decides the bridge construction, always (weakly) improves upon the case with no separation. Interestingly, in this situation a bridge will always be built, since that is the only way for the representative taking this decision to ensure that she will have some local public good on her own jurisdiction.

The possibility of endogenous spillovers sheds a new light into the debate about the relative advantages of decentralization thanks to the cases where the bridge is not built in one of the regimes. If it is not built under centralization, decentralization dominates, irrespective of whether it provides it or not. When the bridge is built only in the centralized regime, decentralization may dominate in some cases. All in all, we interpret our results as suggesting that endogenizing the level of spillovers tilts the debate in favor of decentralization.

The remainder of the paper is organized as follows. Section 2 sets up the model and presents the optimal solution which will be used as a benchmark to the centralized and decentralized systems. Section 3 analyzes the benevolent government approach, while Section 4 is devoted to the political economy perspective, with and without separation of powers at the central level. Section 5 discusses some of our assumptions and concludes.

2 The model

Consider an economy which is divided into 2 geographically distinct districts, indexed by $i \in \{1, 2\}$. Districts have unequal populations, an assumption which allows us to rank the regimes in terms of how they protect minorities.[3] Each district has a continuum of citizens with a mass of $0 < \lambda < 1$ and $1 - \lambda$, respectively. There are 4 goods in the economy: a single private good, x; two local public goods, g_1 and g_2, each one associated with a particular district; and a discrete national public project, G.[4] The local public goods can be anything from parks or museums to education policy, while the national public project is anything that decreases the distance between the two districts, such as bridges or roads to decrease the physical distance, or a language education program to decrease the cultural distance. Hence, while local public goods provide utility directly, the role of the public project is to allow for a better access of the individuals to the neighbor's local public good. We shall henceforth refer to the former as a bridge, keeping in mind that this can be both physical or cultural.

We follow Besley and Coate (2003) and assume that citizens' utility is linear in the private good and logarithmic in the two local public goods.[5] The utility of an individual with preference intensity for public good θ living in jurisdiction i is

$$u_i(x, \mathbf{g}, \kappa; \theta) = x + \theta[\ln(g_i) + \kappa \cdot \ln(g_j)];\; i, j = 1, 2;\; i \neq j \tag{1}$$

where $\mathbf{g} = (g_1, g_2)$ is the vector of local public goods, and $0 \leq \kappa < 1$ measures the degree of spillovers provided by the bridge, or its *quality*, where $\kappa = 0$ when the bridge is not built. Note that we rule out situations where the public good in the other district is more valued by citizens than their own public good. Individuals are heterogeneous regarding their preference intensity for the public good, $0 < \theta \leq \bar{\theta}$, and are endowed with X units of the private good (assumed sufficiently high to fund the implemented

[3]The assumption also avoids the uncertainty effect of the non-cooperative legislature, whose consequences have been extensively studied by Besley and Coate (2003).

[4]As argued by Lockwood (2002), many public goods are actually discrete.

[5]Dur and Roelfsema (2005) and Schnellenbach et al. (2010) also assume quasi-linear utility functions.

levels of the local public goods and the bridge, if built). We assume that the mean and median type, m_i, coincide in each district.[6] Without loss of generality, we order districts such that $m_1 \geq m_2$. Public goods are produced with a linear technology whereby $\rho > 0$ units of the private good are required to produce one unit of the local public good, while the bridge costs $\gamma > 0$ units of the private good. A higher γ may reflect the fact that districts are relatively far from each other, or separated by natural barriers. It may also reflect more distant languages (Ginsburgh et al., 2005). For reasons that will become clear later, we also assume that $\rho \leq (1-\lambda)m_2$.

We study two distinct government regimes, decentralization and centralization. Under decentralization, local public goods are decided and financed locally, by a uniform head tax levied on district residents. Under centralization, local public goods are chosen by the central government and financed by a uniform head tax levied on all citizens.[7] The bridge is always chosen centrally, and financed through a uniform tax levied on all citizens. Accordingly, a resident of district 1 consumes

$$X - \frac{\rho}{\lambda} g_1 - \gamma G \tag{2}$$

and a resident of district 2 consumes

$$X - \frac{\rho}{1-\lambda} g_2 - \gamma G \tag{3}$$

in the decentralized regime, and they both consume

$$X - \rho(g_1 + g_2) - \gamma G \tag{4}$$

in the centralized one, where G is an indicator function which is equal to 1 if the bridge is built, and 0 otherwise.[8]

The optimal benchmark

For later reference, let us find the optimal levels for both local public goods and the optimal bridge provision rule that maximize the weighted sum of utilities. The

[6] We are thus abstracting from distributional considerations.

[7] This approach follows the mainstream in the literature (see Besley and Coate, 2003; Redoano and Scharf, 2004; Dur and Roelfsema, 2005).

[8] Note that $G = 0$ implies a spillover $\kappa = 0$ and $G = 1$ implies a spillover $\kappa \geq 0$.

benevolent social planner solves

$$\max_{g_1, g_2} \quad [\lambda m_1 + (1-\lambda) m_2\, \kappa] \ln(g_1) + [(1-\lambda) m_2 + \lambda m_1\, \kappa] \ln(g_2) - \rho(g_1 + g_2) - \gamma G$$

which yields

$$\big(g_1^o(\kappa), g_2^o(\kappa)\big) = \left(\frac{\lambda m_1 + (1-\lambda) m_2\, \kappa}{\rho}, \frac{(1-\lambda) m_2 + \lambda m_1\, \kappa}{\rho} \right) \tag{5}$$

These provision levels simply reflect the usual trade-off between the marginal benefit and the marginal cost of providing one more unit of the public good. Note that the local public good is at least as high in district 1 than in district 2 if and only if $\lambda m_1 \geq (1-\lambda) m_2$: there is a trade-off between scale and taste for the public good. If the district with a low public good preference is sufficiently more populated, then it is optimal to provide it with a higher public good level. This is a straightforward consequence of Samuelson's rule. We rule out this possibility in our analysis.

Assumption 1.

$$\lambda m_1 \geq (1-\lambda) m_2$$

Observe that this assumption is compatible with any of the jurisdictions being the majoritarian one.

We now discuss the optimality of building the bridge. It turns out that there is a cut-off value of γ above which the bridge should not be built. Let this cut-off be denoted by $\hat{\gamma}^o(\kappa)$ and define

$$W^o(\kappa) = \lambda u_1(x, g_1^o(\kappa), g_2^o(\kappa), \kappa; m_1) + (1-\lambda) u_2(x, g_1^o(\kappa), g_2^o(\kappa), \kappa; m_2) + \gamma G$$

the total welfare of the economy when the local public goods are optimally provided, excluding the provision cost of the bridge. Hence,

$$\begin{aligned}\hat{\gamma}^o(\kappa) = W^o(\kappa) - W^o(0) = {} & [\lambda m_1 + (1-\lambda) m_2\, \kappa] \ln\big(g_1^o(\kappa)\big) - \lambda m_1 \ln\big(g_1^o(0)\big) + \\ & + [\lambda m_1\, \kappa + (1-\lambda) m_2] \ln\big(g_2^o(\kappa)\big) - (1-\lambda) m_2 \ln\big(g_2^o(0)\big) \\ & - \kappa[\lambda m_1 + (1-\lambda) m_2]\end{aligned}$$

Notice that the bridge is not built when it provides no spillovers, *i.e.*, $\hat{\gamma}^o(0) = 0$. Also,

$$\frac{\mathrm{d}\hat{\gamma}^o(\kappa)}{\mathrm{d}\kappa} = \frac{\mathrm{d}W^o(\kappa)}{\mathrm{d}\kappa} = \lambda m_1 \ln\left(g_2^o(\kappa)\right) + (1-\lambda)m_2 \ln\left(g_1^o(\kappa)\right) \tag{6}$$

which is positive, due to Assumption 1 and the fact that $\rho \leq (1-\lambda)m_2$. Hence, the range of costs for which the bridge is built increases with the spillover it provides. Also, given that the bridge is built, total welfare increases in its quality. This stems from the fact that the building cost is independent of the bridge's quality.

3 Benevolent governments and uniform provision at the central level

In this section, we take up on Oates' (1972) approach, and model utility-maximizing governments.

3.1 The decentralized regime

Working by backwards induction, we let jurisdictions decide simultaneously and independently the provision of local public goods in the second stage, given the decision to build the bridge in the first. Using (1) and (2), the local government in jurisdiction 1 maximizes

$$m_1[\ln(g_1) + \kappa \ln(g_2)] - \frac{\rho}{\lambda}g_1 - \gamma G$$

The local government in jurisdiction 2 proceeds analogously, where (3) replaces (2). It is then straightforward to obtain the outcome

$$\left(g_1^d, g_2^d\right) = \left(\frac{\lambda m_1}{\rho}, \frac{(1-\lambda)m_2}{\rho}\right) \tag{7}$$

Hence, the local public goods do not depend on whether or not the bridge is built. In the first stage, the central government decides upon the bridge construction, anticipating that local public goods will be given by (7). The threshold $\hat{\gamma}^d(\kappa)$ is then given by

$$\hat{\gamma}^d(\kappa) = W^d(\kappa) - W^d(0) = \lambda m_1\, \kappa \ln\left(g_2^d\right) + (1-\lambda)m_2\, \kappa \ln\left(g_1^d\right)$$

where $W^d(\kappa)$ is defined analogously to $W^o(\kappa)$. Again, $\hat{\gamma}^d(0) = 0$ and moreover

$$\frac{\mathrm{d}\hat{\gamma}^d(\kappa)}{\mathrm{d}\kappa} = \frac{\mathrm{d}W^d(\kappa)}{\mathrm{d}\kappa} = \lambda m_1 \ln\left(g_2^d\right) + (1-\lambda) m_2 \ln\left(g_1^d\right) > 0 \tag{8}$$

Note that local policy-makers fail to internalize the spillover effects originated by the bridge and hence $g_i^d \leq g_i^o(\kappa)$, as revealed by the comparison of (5) and (7). As a result, $\hat{\gamma}^d(\kappa) < \hat{\gamma}^o(\kappa)$, *i.e.* there is an intermediate range of γ for which the bridge is not built when it is optimal to do so.[9]

3.2 The centralized regime

The central government chooses a uniform supply of local public goods, $g_1 = g_2 = g$. Using (1) and (4), the central government maximizes

$$[\lambda m_1 + (1-\lambda) m_2](1+\kappa)\ln(g) - \rho g - \gamma G$$

The chosen level of local public good is then

$$g^c(\kappa) = \frac{\lambda m_1 + (1-\lambda) m_2}{2\rho}(1+\kappa) \tag{9}$$

As usual, the centralized solution has the advantage that it internalizes the spillover effect, at the cost of not taking into account taste heterogeneity. Using (9), it is obvious that the threshold cost of the bridge above which it is not built is given by

$$\begin{aligned}\hat{\gamma}^c(\kappa) = W^c(\kappa) - W^c(0) &= [\lambda m_1 + (1-\lambda) m_2](1+\kappa)\ln\left(g^c(\kappa)\right) \\ &- [\lambda m_1 + (1-\lambda) m_2]\ln\left(g^c(0)\right) - \kappa[\lambda m_1 + (1-\lambda) m_2]\end{aligned}$$

where $W^c(\kappa)$ has the usual meaning. As expected, if the bridge provides no spillovers, then it is not built ($\hat{\gamma}^c(0) = 0$). Moreover,

$$\frac{\mathrm{d}\hat{\gamma}^c(\kappa)}{\mathrm{d}\kappa} = \frac{\mathrm{d}W^c(\kappa)}{\mathrm{d}\kappa} = [\lambda m_1 + (1-\lambda) m_2]\ln\left(g^c(\kappa)\right) > 0 \tag{10}$$

which states that the range of costs for which the bridge is build increases with its quality. In the next subsection we discuss which regime dominates in terms of total

[9] This result is proved in Lemma 1 below.

welfare.

3.3 Comparing the two regimes

In Oates (1972) and Besley and Coate (2003), if preferences are homogeneous, that is, $m_1 = m_2$, the centralized regime coincides with the optimum. This is no longer the case here, for the population size creates an additional source of heterogeneity between the two districts. Hence, the centralized regime coincides with the optimum if and only of $\lambda m_1 = (1-\lambda)m_2$, *i.e.*, if the *total willingness to pay* for the public good is the same in the two jurisdictions. If districts are heterogeneous, simple algebra allows one to show that $g_2^d < g^c(\kappa)$. However, we cannot compare g_1^d to $g^c(\kappa)$. Indeed, while the decentralized regime takes into account the higher taste for the public good in jurisdiction 1, the centralized regime incorporates the fact that the residents of district 2 also benefit from g_1. Hence, $g^c(\kappa) > g_1^d$ if and only if

$$\kappa > \frac{\lambda m_1 - (1-\lambda)m_2}{\lambda m_1 + (1-\lambda)m_2} > 0$$

i.e, the degree of spillovers must be sufficiently higher than inter-jurisdictional heterogeneity.

The welfare comparison hinges on whether the bridge is built under each regime. However, since the bridge gives access to different levels of local public goods under each regime, the decision to build it is not enough *per se* to rank welfare levels. The next two results treat each of these issues in turn.

Lemma 1. *The decision to build the bridge is characterized by a cut-off cost level in both the decentralized and the centralized regime, such that the bridge is built if the construction cost is below the cut-off level. The cut-off cost of the decentralized regime is lower, while that of the centralized regime is higher, than the first best level.*

Proof. See Appendix A. □

Notice that the decision rule regarding the bridge construction is the same under both regimes, and it coincides with the first best one. Hence the threshold cost changes exclusively due to the provision of local public goods, which differs across the three scenarios. In the decentralized regime, local public goods are under-provided because only the local benefit is taken into account, thus decreasing the benefit from building the bridge. Under the centralized regime, local public goods are provided according to the average preference across jurisdictions, implying that there is under-provision in the high-taste district 1 and over-provision in the low-taste district 2. The interaction between these provision levels ultimately determines whether the bridge is built or not. Not surprisingly, the second effect dominates and the bridge provides a higher benefit in the centralized regime than under the first best scenario.

Armed with the threshold comparison in Lemma 1, we now look at the welfare ranking across regimes.

Proposition 1. *There is a threshold value of the bridge building cost, $\tilde{\gamma}$, above which decentralization always dominates centralization. When the building cost is lower than the threshold, decentralization dominates only if the bridge quality is sufficiently low. Moreover, $\tilde{\gamma} < \hat{\gamma}^c(\kappa)$, i.e., it may happen that decentralization dominates centralization when the bridge is built under the latter, and not the former.*

Proof. See Appendix B. □

What does this result add to our knowledge about the trade-off between centralization and decentralization? In the classical exogenous spillover case, centralization dominates when the degree of spillovers is high enough. This is also what one obtains here for sufficiently low construction costs, when both regimes build the bridge. It is also not surprising that decentralization is the dominating regime when building costs are so high that there is no bridge in either regime (this is just equivalent to the no spillover case). Interestingly, however, there are cases in which the centralized regime provides the bridge while the decentralized one does not, and the latter dominates the former independently of the bridge's quality. This is a consequence of the fact that the centralized regime provides the bridge when it is not optimal to do so. We may thus argue that making spillovers endogenous improves the case for a decentralized regime. Moreover, Besley and Coate (2003) show that, when districts are identical and spillovers are absent, the two regimes generate the same level of surplus, which is not the case here. When spillovers are absent, irrespective of the two districts' preferences for the public good, no regime provides the bridge and hence decentralization is always the preferred one, since it caters for local tastes. Also, and as mentioned previously, the coincidence between the centralized regime and the optimum now arises only when $\lambda m_1 = (1-\lambda)m_2$. This is a natural consequence of our introduction of size heterogeneity across jurisdictions.

4 Majoritarian elections

We now model political decision making, based on the citizen-candidate frameworks proposed in Besley and Coate (1997) and Osborne and Al Slivinski (1996). We follow Besley and Coate (1997) in assuming that decision makers cannot commit to a given policy platform prior to the election stage, and therefore they always follow their preferred policies once in office. Hence, citizens elect policy-makers whose preferences match the ones they like. Policy preferences are common knowledge, and there is no cost of entering the political market.

4.1 The decentralized regime

Under decentralization, the bridge construction is decided by the national legislature, while elected regional representatives are responsible for setting the supply of local public goods in each jurisdiction. All representatives are elected by majority voting. We set up a four stage process, consisting of: *(i)* citizens in each jurisdiction elect a legislator to the national legislature, *(ii)* the legislature decides whether to build the bridge, *(iii)* citizens in each jurisdiction elect a policy-maker to the regional government, and, finally, *(iv)* local policy-makers in each jurisdiction choose simultaneously and independently the supply of local public goods.[10]

As usual, we look for a Subgame Perfect Nash Equilibrium. Let the type of the elected policy-maker in stage 3 in district i be given by θ_i, $i = 1, 2$. Working by backwards induction, and recalling that elected citizens follow their preferred policies when in office, it is straightforward to use (1), (2) and (3) to obtain the provided level of local public goods

$$(g_1, g_2) = \left(\frac{\lambda\theta_1}{\rho}, \frac{(1-\lambda)\theta_2}{\rho}\right)$$

In the third stage, citizens vote to elect the local policy-maker. Following Besley and Coate (2003), a pair of representative types (θ_1^*, θ_2^*) is majority preferred under decentralization if, in each district i, a majority of citizens prefers the type of their representative to any other type $\theta \in (0, \bar{\theta}]$, given the type of the other district's

[10] Considering simultaneous elections for the national and regional governments followed by simultaneous decisions in a second stage would keep our results unchanged. By contrast, having local decisions before the national ones would create strategic interaction at the local level. In this case, local policy-makers in one region can use local public goods to influence the level of spillovers implemented by the legislature, and these depend on the decision of the other region.

representative θ_j, $j \neq i$. Using (1) and (2), a citizen of type θ in district 1 enjoys a level of surplus of (an analogous expression holds for a district 2 resident)

$$\theta \left[\ln\left(\frac{\lambda\theta_1}{\rho}\right) + \kappa \ln\left(\frac{(1-\lambda)\theta_2}{\rho}\right) \right] - \theta_1 - \gamma G$$

As these preferences are clearly single-peaked, the median voter theorem implies that the elected pair is (m_1, m_2), thus local public good provision coincides with the traditional approach and is given by (7).

Let us now look at the bridge construction stage. At this point, it is necessary to define the rules of how the legislature behaves. There is no unified approach in the literature, although a number of different alternatives have been suggested (see Lockwood, 2006). Here, we will use a slightly modified version of the closed rule legislative bargaining proposed in a seminal paper by Baron and Ferejohn (1989), and used in Besley and Coate (2003). In stage 2, the legislature makes a proposal regarding whether the bridge should be built, which must then find the support of a minimum coalition. In our two jurisdiction model, the minimum winning coalition is the representative of the jurisdiction with the greatest mass of residents. In the first stage, citizens in each jurisdiction elect the legislator who will take part in the national legislature. As the representative which sets the policy is the one from the most populated district, it is straightforward to show that the median voter of this jurisdiction will be the decision taker at the national level. When district 1 is the most populated one, use (1) and (2) to obtain that she decides to build the bridge when

$$m_1 \left[\ln\left(\frac{\lambda m_1}{\rho}\right) + \kappa \ln\left(\frac{(1-\lambda)m_2}{\rho}\right) \right] - m_1 - \gamma \geq m_1 \ln\left(\frac{\lambda m_1}{\rho}\right) - m_1$$

yielding a cut-off cost level of

$$\hat{\gamma}^{pd1}(\kappa) = m_1\, \kappa \ln\left(\frac{(1-\lambda)m_2}{\rho}\right)$$

where the superscript $pd1$ stands for the political economy decentralized regime, when the median voter of jurisdiction 1 is the decision maker.

Given that the local public good levels coincide both under the traditional and the political economy approach, the difference in the cut-off bridge costs is due to the different decision rule. It is instructive to understand the differences between this cut-off cost level and $\hat{\gamma}^d$. Firstly, the median voter benefits from the bridge but does not bear its full cost, while the planner takes into account the full bridge cost when deciding upon construction. Secondly, the impact of the bridge on the residents of district 2 is ignored. The first effect leads $\hat{\gamma}^{pd1}$ to be higher, while the second effect leads it to be smaller, than $\hat{\gamma}^d$, so we cannot compare the two, *a priori*. The two effects clearly appear when we compute the difference between the two cut-off cost levels

$$\hat{\gamma}^{pd1} - \hat{\gamma}^d = (1-\lambda)\kappa \left[m_1 \ln\left(\frac{(1-\lambda)m_2}{\rho}\right) - m_2 \ln\left(\frac{\lambda m_1}{\rho}\right) \right]$$

Analogously, when district 2 is the majoritarian district

$$\hat{\gamma}^{pd2}(\kappa) = m_2\, \kappa \ln\left(\frac{\lambda m_1}{\rho}\right)$$

and the same two effects are present, so that we cannot *a priori* compare $\hat{\gamma}^{pd2}$ with $\hat{\gamma}^d$.

4.2 The centralized regime

Under centralization, both local public goods and the bridge construction are decided by the national legislature, and all costs are split uniformly across jurisdictions. The order of events is as follows. Firstly, citizens in each jurisdiction elect a representative (delegate) among them, by majority voting. Thereafter, the legislature chooses the policy vector. Again, it is a straightforward exercise to show that the median voter of the most populated district will decide the policy vector. The provided level of local public goods is given by

$$\left(g_1^{pc1}, g_2^{pc1}(\kappa)\right) = \left(\frac{m_1}{\rho}, \frac{\kappa\, m_1}{\rho}\right) \tag{11}$$

if district 1 is the most populated (*i.e.*, $\lambda > 1/2$), and

$$\left(g_1^{pc2}(\kappa), g_2^{pc2}\right) = \left(\frac{\kappa\, m_2}{\rho}, \frac{m_2}{\rho}\right)$$

otherwise. Clearly, the own jurisdiction local public good is higher in the centralized regime because the cost is shared equally across the two jurisdictions, *i.e.*, the median voter of, say, district 1, pays ρg_1 here and $\rho g_1/\lambda$ in the decentralized regime. Secondly, the decision-maker decides the amount of public good in the neighbor jurisdiction as well, which naturally depends on the bridge quality. This effect compounds with the above common pool one, so that we cannot *a priori* rank public good levels in the neighbor jurisdiction across the two regimes. A third effect stems from the different preferences reflected in the decision about the local public good. Not surprisingly, it takes a sufficient quality, relative to the common pool effect and the preference heterogeneity, for the local public good in the neighbor jurisdiction to be higher in the centralized system, *i.e.* $g_2^{pc1} > g_2^{pdi}$, $i = 1, 2$ when $\kappa > (1-\lambda)m_2/m_1$ and $g_1^{pc2} > g_1^{pdi}$, $i = 1, 2$ when $\kappa > \lambda m_1/m_2$. The interested reader will note that Besley and Coate's (2003) uncertainty effect is not present here, due to our assumption of unequal jurisdiction sizes. However, our framework borrows from theirs the misallocation and common pool effects, which drive our analysis.

The ranking of the neighbor local public goods determines the decision about the bridge. If district 1 is the majoritarian one, using (1) and (4), the median voter decides to build the bridge if

$$m_1\left[\ln\left(\frac{m_1}{\rho}\right) + \kappa\ln\left(\frac{\kappa\, m_1}{\rho}\right)\right] - m_1(1+\kappa) - \gamma \geq m_1\ln\left(\frac{m_1}{\rho}\right) - m_1$$

yielding a cut-off cost level of

$$\hat{\gamma}^{pc1}(\kappa) = \kappa\, m_1\left[\ln\left(\frac{\kappa\, m_1}{\rho}\right) - 1\right]$$

Analogously, when the majority of the population lives in jurisdiction 2,

$$\hat{\gamma}^{pc2}(\kappa) = \kappa\, m_2\left[\ln\left(\frac{\kappa\, m_2}{\rho}\right) - 1\right]$$

4.3 Comparing the two regimes

As with the benevolent planner case, the welfare difference across the two regimes hinges on two factors: firstly, whether or not the bridge is built and, secondly, the different local public good levels under each scenario. While it is possible to compare the cut-off costs with the first best one, the exercise is less instructive here, for the decision about the bridge construction now follows a different rule (which is not the case with the benevolent planner approach). We therefore proceed by comparing the cut-off cost levels across the two regimes. Figure 1 displays the comparison of the two cut-off levels in the $(\lambda, m_1/m_2)$ space. Its main findings are summarized in Lemmas 2 and 3.

Lemma 2. *When the high-taste district is majoritarian, the decision to build the bridge is characterized by a cut-off cost level. The cut-off cost level under centralization is always lower than that of the decentralized regime, except when districts are very heterogeneous (or district 1 is sufficiently more populated) and the bridge quality is sufficiently high.*

Proof. See Appendix C. □

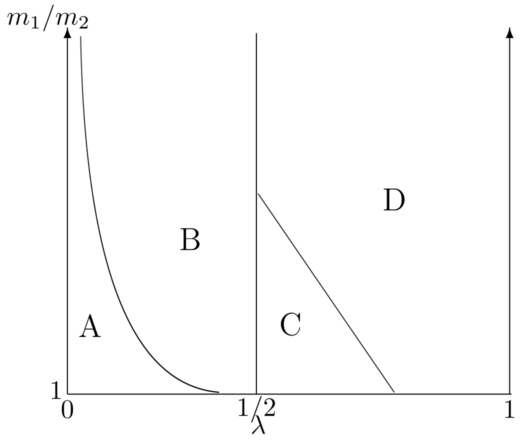
The intuition for this result is quite simple. The benefit of the bridge in both regimes is related to the level of local public good in jurisdiction 2. In the centralized regime building the bridge implies providing some public good in jurisdiction 2, which entails a cost since the budget is centralized. This cost is outweighed by the benefit only when the public good in jurisdiction 2 is sufficiently higher in the centralized regime than in the decentralized one: that requires both a high degree of spillovers and a high public good taste of 1's median voter *vis-a-vis* that of 2's (or a sufficiently populated district 1). These effects explain the cut-off bridge quality above which the centralized regime builds more often the bridge, given by

$$\kappa > \frac{(1-\lambda)m_2}{m_1}e \tag{12}$$

We now look at the case where the majority lives in district 2.

Lemma 3. *When the low-taste district is majoritarian, the decision to build the bridge is characterized by a cut-off cost level. The cut-off cost level under centralization is always lower than that of the decentralized regime, except when districts are not too heterogeneous, district 2 is sufficiently more populated, and the bridge quality is sufficiently high.*

Proof. See Appendix D. □



Legend

A $\hat{\gamma}^{pc2}(\kappa) < \hat{\gamma}^{pd2}(\kappa)$ when $\kappa < \frac{\lambda m_1 e}{m_2}$
B $\hat{\gamma}^{pc2}(\kappa) < \hat{\gamma}^{pd2}(\kappa)$ for all κ
C $\hat{\gamma}^{pc1}(\kappa) < \hat{\gamma}^{pd1}(\kappa)$ for all κ
D $\hat{\gamma}^{pc1}(\kappa) < \hat{\gamma}^{pd1}(\kappa)$ when $\kappa < \frac{(1-\lambda) m_2 e}{m_1}$

Figure 1: Ranking of cut-off levels for the bridge cost, $\left(\hat{\gamma}^{pdi}(\kappa), \hat{\gamma}^{pci}(\kappa)\right)$, $i = 1, 2$.

The reasoning here is analogous to Lemma 2, except that the local public good in jurisdiction 1 is sufficiently high to overcome the increased cost under centralization when the bridge quality is high enough and the preference of district 2's median voter is sufficiently high, that is, districts are not too heterogeneous. The combination of these effects yields the following cut-off level of κ, above which $\hat{\gamma}^{pc2}(\kappa) > \hat{\gamma}^{pd2}(\kappa)$

$$\kappa > \frac{\lambda m_1}{m_2} e \tag{13}$$

Figure 1 also highlights an interesting contrast between the two cases. While when district 1 is the majoritarian one, there exists a κ above which $\hat{\gamma}^{pc1} > \hat{\gamma}^{pd1}$ for all possible values of λ, this is not true when district 2 is majoritarian. When $\lambda > 1/e$, we always have $\hat{\gamma}^{pc2} < \hat{\gamma}^{pd2}$, irrespective of the degree of spillovers created by the bridge. This is a natural consequence of the high local public good provided in jurisdiction 1 under the decentralized regime when λ gets sufficiently close to $1/2$.

Armed with the threshold comparison in Lemmas 2 and 3, we may now address the welfare comparison across the two regimes. We tackle this issue in two separate propositions.

Proposition 2. *Decentralization dominates centralization when*

(i) both regimes build the bridge and the degree of spillovers provided by the bridge is not too high;

(ii) the centralized regime does not build the bridge.

Proof. See Appendix E. □

When the bridge is built under the two regimes, one finds a similar result to the traditional approach one, namely, that centralization dominates when the bridge is sufficiently good. While the decentralized provision of local public goods coincides under both the traditional and the political economy approach, the centralized one is very different across the two decision rules. However, it is still true that spillovers are taken into account, albeit for different reasons. Here, the median voter of the majoritarian jurisdiction decides upon the level of local public good in the neighbor jurisdiction, which she can enjoy only if the bridge is built. However, the own-jurisdiction local public good provision does not vary with the level of spillovers. It is also interesting to note that preference heterogeneity is not reflected in the local provision in the political economy setup, as much as in the traditional approach. Again, this happens for a very different reason. Instead of a benevolent planner obeying a constitutional rule of uniform provision, we have the majority deciding the local public good of the minority according to the tastes of the former. Majoritarian elections are thus a way to introduce the same effects that we find in the traditional approach *à la* Oates, while escaping the *ad hoc* assumption of uniform provision.[11]

The outcome where the bridge is not built under the centralized system performs very badly in terms of welfare. This is because the majority will only provide any public good in the minoritarian jurisdiction if there is a bridge that allows the majority to enjoy it. Hence, with no bridge the minority finds itself with no local public good and no possibility to enjoy the neighbor's. While it can be argued that this sharp result is due to the logarithmic assumption, which drives the payoff of the minoritarian jurisdiction to an arbitrarily large negative value, the intuition is very clear and it is obvious that any utility function would allow us to build a very strong case against centralization when this regime decides not to build the bridge.

The above proposition treats all possible cases when the bridge quality is below the thresholds defined in (12) and (13). Should the bridge quality be above the thresholds, however, there is an intermediate range of bridge costs for which the bridge is built under centralization and not decentralization. Such cases are the object of the next proposition. Note, however, that when $\gamma > \gamma^{pci}$, $i = 1, 2$, the centralized regime does not provide the bridge, and the above result that decentralization dominates applies.

[11] As Besley and Coate (2003) have also pointed out.

Proposition 3. *When only the centralized regime builds the bridge, if the low-taste district is majoritarian, decentralization never dominates. If the high-taste district is majoritarian, then decentralization dominates centralization except, possibly, if districts are sufficienty homogeneous.*

Proof. See Appendix F. □

Let us start with the case where district 2 is majoritarian. What is the impact, for the welfare of district 2, of going from the centralized regime with a bridge to the decentralized one without? On the one hand, the local public good is higher under the former, due to the common pool effect. On the other hand, this comes at a cost (increased local public good and bridge bill). However, the median resident of this jurisdiction decides to build the bridge precisely because its benefit pays the increased cost. Hence, only the own-local public good effect remains, and it is actually straightforward to show that, even in the worst case scenario where the bridge costs $\hat{\gamma}^{pc2}$, district 2's residents gain $-(1-\lambda)m_2\ln(1-\lambda)$ when going from the decentralized to the centralized regime. And what about district 1? Firstly, the local public good is higher under centralization, given that the bridge quality is very high (otherwise the situation where only the centralized regime provides the bridge never arises). Secondly, there is the possibility to cross the bridge and enjoy the local public good in district 2. Thirdly, the total fiscal bill changes, but not proportionally to the public good increase, due to the common pool of the centralized budget. On the other hand, the benefit that the median voter of jurisdiction 2 derives from the bridge ($m_2\,\kappa\ln(\kappa m_2/\rho)$) is lower than the one derived by the district 1's median resident ($m_1\,\kappa\ln(m_2/\rho)$). In other words, the willingness to pay for the bridge is higher for this latter, explaining why she enjoys a higher welfare level in the centralized regime, with the bridge. Indeed, again in the worst case scenario under centralization, district 1's residents gain

$$\lambda\left[m_1\left(\ln\left(\frac{\kappa\,m_2}{\lambda m_1}\right)+\kappa\ln\left(\frac{m_2}{\rho}\right)\right)-m_2+m_1-m_2\,\kappa\ln\left(\frac{\kappa\,m_2}{\rho}\right)\right]$$

which is positive, noticing that the second term is always higher than the last (in absolute terms).[12]

[12]Please refer to Appendix F and, in particular, (21).

It is now easier to understand why this need not be the case when the majority lives in district 1. While it is still true that this district's residents gain from moving into a centralized system, district 2's residents need not have a higher willingness to pay for the bridge. They value the bridge at $m_2\,\kappa \ln(m_1/\rho)$, while the district 1's residents value it at $m_1\,\kappa \ln(m_1\kappa/\rho)$, two values which cannot, *a priori*, be ranked. Centralization may dominate if districts are sufficiently homogeneous, *i.e.*, when district 2's residents have a strong preference for the public good. To understand why, notice that when the bridge quality is sufficiently high that the bridge is built under centralization, but not decentralization, then it is also true that the local public good in district 2 is higher in the centralized regime. Given the quality of the bridge and m_1's high taste for the public good, she ends up providing a higher local public good in jurisdiction 2 than what m_2 would do herself. From the viewpoint of jurisdiction 2's residents, centralization has two advantages: *(i)* it provides a higher level of local public good, and *(ii)* it allows them to enjoy the neighbor jurisdiction's public good since a bridge is provided in the centralized regime. This is all the better news for district 2's residents the more they enjoy the public good, that is, the higher is m_2.

4.4 Centralization with separation of powers: protecting minorities

It has been recognized in the literature that the minimum winning coalition view of legislative decision-making is the exception rather than the rule (Besley and Coate, 2003). In reality, most economies organize decision-making under less radical decision rules, which usually imply some sort of separation of powers among the distinct members that compose the legislature. We now modify our setup and suppose that, in the centralized regime, one of the local representatives decides upon local public good provision, whereas the other takes the decision regarding the bridge.

It is again a straightforward exercise to show that the median voter is pivotal in each jurisdiction. Suppose that the local representative of district 1 decides local public good provision. Then, local public goods are given by (11). When the local representative of the other district is called upon to take the decision regarding the bridge, she decides to build it if and only if

$$m_2\left[\ln\left(\frac{m_1\,\kappa}{\rho}\right)+\kappa\ln\left(\frac{m_1}{\rho}\right)\right]-(m_1+m_1\,\kappa)-\gamma\geq m_2\ln(0)-m_1$$

from which it is obvious that the bridge is always built. This happens because if the representative of district 2 decides against the bridge construction, then she finds herself with no local public good. Hence, it is always in her interest to build the bridge so as to give the right incentives to the representative of district 1 to provide a positive level of public good in her jurisdiction. Naturally, this happens also when the roles are reversed, *i.e.*, when the representative of jurisdiction 2 decides local public goods and that of jurisdiction 1 decides upon the bridge construction. This allows us to state our next result.

Proposition 4. *In a centralized system with separation of powers, the bridge is built for any finite building cost. A centralized regime with separation of powers weakly dominates that with no separation, in terms of total welfare.*

Proof. The first part follows from the discussion above. As regards the second part, notice that when the bridge is built in both regimes, then total welfare is the same in both. However, when the bridge is built under separation and not under no separation, total welfare is infinitely low in this latter, for the residents of the minority jurisdiction find themselves with no local public good. □

The message here is clear. If the regime is to be centralized, then it should allocate different decisions to different constituents. This system enables the minority to effectively force the majority to build some positive level of public good in the minoritarian district. It is actually a very simple and powerful device against the very negative welfare outcome of the centralized regime (with no separation of powers) when the bridge cost is high enough such that this latter is not built. How does this separation of powers scenario compare with decentralization? Proposition 3 and the first part of Proposition 2 apply straightforwardly, since they refer to situations in which the centralized regime without separation of powers builds the bridge, hence attaining the same welfare level as with separation. Hence, we have that decentralization dominates when it builds the bridge and its quality is low, or when it does not, if the high-taste district is majoritarian and districts are sufficiently heterogeneous.

5 Conclusion

This paper highlights how endogenizing the level of spillovers changes our usual wisdom about the trade-off between centralization and decentralization. The spillovers are provided by a national public good (the bridge) which decreases the (geographical or cultural) distance between two heterogeneous districts, which can be built at a cost. Depending on the exogenous quality of the bridge, the residents of a given district may enjoy a lower or higher share of the neighbor's local public good.

We analyze both a traditional benevolent government *à la* Oates with the requirement of uniform provision at the central level, and a political economy setting where decisions are taken by majoritarian elections. The endogenous spillovers framework generates three different ranges in the space of the bridge cost (which depend on the decision making rule and on which district hosts the majority of the citizens). When the cost is low, the bridge is built under both regimes. When the cost is intermediate, one regime builds the bridge while the other does not and, for sufficiently high costs, the bridge is not built under both regimes. We show that in the first (low cost) range the usual insight that centralization dominates when spillovers are high drives the results. In the third (high cost) range decentralization always dominates.

In the intermediate cost range, it depends on which regime builds the bridge. In the traditional approach, it is always centralization that builds it, and decentralization dominates when the bridge cost is above a certain threshold. This is because building the bridge is costly, and there are cases in which the centralized regime builds the bridge while it is not optimal to do so, *i.e.*, the benefits are outweighed by the building cost. In the political economy approach, if the centralized regime is not providing the bridge, we obtain the worst possible global welfare, for the minoritarian district finds itself with no local public good (the majority does not provide it with no bridge to enjoy it), and no possibility to enjoy the majoritarian district's one. When it is the centralized regime that provides the bridge, then it may still happen that decentralization is preferred when the majority of the population resides in the high-taste district. Finally, we show that the very negative outcome under the centralized regime can be overcome by a simple, yet very powerful, mechanism: separation of powers. Allocating the decision right over the bridge construction to the minority ensures that the bridge is always built and that they no longer find themselves in the no-own public good, no-bridge scenario.

Our results were obtained using a number of standard assumptions: quasi-linear utility function and provision costs measured in units of the numéraire. More importantly, we have assumed that the benefit provided by local public goods is logarithmic. While some intermediate steps in our results would be possible to obtain under a general utility function (some comparisons of local public good levels and threshold bridge costs), the full-fledged welfare comparison we provide would not be possible to obtain. Finally, the logarithmic utility has the nice property that zero provision drives the utility to an arbitrarily large negative level. This, combined with the quasi-linear assumption, generates a quite pessimistic scenario regarding the welfare cost that the majority may impose upon the minority when it concentrates all the decision power. Finally, we assume that the quality of the bridge is exogenous and that the provision costs are invariant with the decision regime. While the latter assumption is just a natural one to make to confer neutrality to our model, the former is a natural direction for future research.

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References

Akai, N. and K. Mikami, "Fiscal decentralization and centralization under a majority rule: A normative analysis," *Economic Systems* 30 (2006), 41–55.

Amir, R., I. Evstigneev and J. Wooders, "Noncooperative versus cooperative R&D with endogenous spillover rates," *Games and Economic Behavior* 42 (2003), 183–207.

Bardhan, P. and D. Mokherjee, "Capture and governance at local and national levels," *The American Economic Review* 90 (2000), 135–139.

Baron, D. and J. Ferejohn, "Bargaining in legislatures," *American Political Science Review* 83 (1989), 1181–1206.

Belleflamme, P. and J. Hindriks, "Yardstick competition and political agency problems," *Social Choice and Welfare* 24 (2005), 155–169.

Besharov, G., "Influence costs in the provision of local public goods," *Duke University, Economics Working Paper No. 01-02* (2002).

Besley, T. and S. Coate, "An economic model of representative democracy," *The Quarterly Journal of Economics* 112 (1997), 85–114.

———, "Centralized versus decentralized provision of local public goods: A political economy approach," *Journal of Public Economics* 87 (2003), 2611–2637.

Bloch, F. and U. Zenginobuz, "The effect of spillovers on the provision of local public goods," *Review of Economic Design* 11 (2007), 199–216.

Calderón, C. and A. Chong, "Volume and quality of infrastructure and the distribution of income: An empirical investigation," *Review of Income and Wealth* 50 (2004), 87–106.

Canning, D., "A database of world infrastructure stocks, 1950-95," *World Bank Policy Research Working Paper No. 1929* (1998).

Cattaneo, A. and R. Winkelmann, "Earnings differentials between German and French speakers in Switzerland," *Swiss Journal of Economics and Statistics (SJES)* 141 (2005), 191–212.

Dur, R. and H. Roelfsema, "Why does centralization fail to internalise policy externalities?," *Public Choice* 122 (2005), 395–416.

Gersbach, H. and A. Schmutzler, "Endogenous spillovers and incentives to innovate," *Economic Theory* 21 (2003), 59–79.

Ginsburgh, V., I. Ortuno-Ortin and S. Weber, "Disenfranchisement in linguistically diverse societies: The case of the European Union," *Journal of the European Economic Association* 3 (2005), 946–965.

Hindriks, J. and B. Lockwood, "Centralization and political accountability," *CORE Discussion Paper No. 2004/52* (2004).

Koethenbuerger, M., "Revisiting the "Decentralization Theorem" – On the role of externalities," *Journal of Urban Economics* 64 (2008), 116–122.

Koppel, O., "Reviewing Oates' insights in a repeated game setting," *Economics of Governance* 6 (2005), 229–243.

Limão, N. and A. Venables, "Infrastructure, geographical disadvantage, transport costs, and trade," *The World Bank Economic Review* 15 (2001), 451.

Lockwood, B., "Distributive politics and the costs of centralization," *The Review of Eco-*

nomic Studies 69 (2002), 313–337.

———, Fiscal decentralization: A political economy perspective, *In: The Handbook of Fiscal Federalism* (Edward Elgar Press, 2006).

Mohanty, A., *Multilingual education – A bridge too far?* In: Multilingual education for social justice: Globalising the local. Mohanty, A.K., Panda, M., Phillipson, R. and Skutnabb-Kangas, T. (eds.) (New Delhi: Orient Blackswan, 2009), 5–17.

Oates, W., *Fiscal Federalism* (Harcourt Brace, New York, 1972).

Osborne, M. and Al Slivinski, "A model of political competition with citizen-candidates," *The Quarterly Journal of Economics* 111 (1996), 65–96.

Piga, C. and J. Poyago-Theotokyb, "Endogenous R&D spillovers and locational choice," *Regional Science and Urban Economics* 35 (2005), 127–139.

Redoano, M., "Does centralization affect the number and size of lobbies?," *Journal of Public Economic Theory* (2010), forthcoming.

Redoano, M. and K. Scharf, "The political economy of policy centralization: Direct versus representative democracy," *Journal of Public Economics* 88 (2004), 799–817.

Rubinchik-Pessach, A., "Can decentralization be beneficial?," *Journal of Public Economics* 89 (2005), 1231–1249.

Schnellenbach, J., L. Feld and C. Schaltegger, "The impact of referendums on the centralisation of public goods provision: A political economy approach," *Economics of Governance* 11 (2010), 3–26.

Strumpf, K., "Does government decentralization increase policy innovation?," *Journal of Public Economic Theory* 4 (2002), 207–241.

Tiebout, C., "A pure theory of local expenditures," *Journal of Political Economy* 64 (1956), 416–424.

Tommasi, M. and F. Weinschelbaum, "Centralization vs. decentralization: A principal-agent analysis," *Journal of Public Economic Theory* 9 (2007), 369–389.

Williams, A., "The optimal provision of public goods in a system of local government," *Journal of Political Economy* 74 (1966), 18–33.

Appendices

A Proof of Lemma 1

Remember that $\hat{\gamma}^o(0) = \hat{\gamma}^d(0) = \hat{\gamma}^c(0) = 0$. We now show that $\mathrm{d}(\hat{\gamma}^o(\kappa) - \hat{\gamma}^d(\kappa))/\mathrm{d}\kappa > 0$, $\kappa > 0$ and $\mathrm{d}(\hat{\gamma}^o(\kappa) - \hat{\gamma}^c(\kappa))/\mathrm{d}\kappa < 0$, $\kappa > 0$, which proves the result.

For the first part, use (6) and (8) to obtain

$$\frac{\mathrm{d}\left(\hat{\gamma}^o(\kappa) - \hat{\gamma}^d(\kappa)\right)}{\mathrm{d}\kappa} = \lambda m_1 \ln\left(\frac{(1-\lambda)m_2 + \lambda m_1 \kappa}{(1-\lambda)m_2}\right) + \\ + (1-\lambda)m_2 \ln\left(\frac{\lambda m_1 + (1-\lambda)m_2 \kappa}{\lambda m_1}\right)$$

which is clearly greater than 0 for any $\kappa > 0$. For the second part, use (6) and (10) to obtain

$$\frac{\mathrm{d}\big(\hat{\gamma}^o(\kappa) - \hat{\gamma}^c(\kappa)\big)}{\mathrm{d}\kappa} = \lambda m_1 \ln\left(\frac{(1-\lambda)m_2 + \lambda m_1 \kappa}{\lambda m_1 + (1-\lambda)m_2}\frac{2}{1+\kappa}\right) + \\ + (1-\lambda)m_2 \ln\left(\frac{\lambda m_1 + (1-\lambda)m_2 \kappa}{\lambda m_1 + (1-\lambda)m_2}\frac{2}{1+\kappa}\right) = \delta(\kappa)$$

It is straightforward to see that $\delta(1) = 0$. Furthermore,

$$\frac{\mathrm{d}\delta(\kappa)}{\mathrm{d}\kappa} = \frac{1}{1+\kappa}[\lambda m_1 - (1-\lambda)m_2]\left(\frac{\lambda m_1}{(1-\lambda)m_2 + \lambda m_1 \kappa} - \frac{(1-\lambda)m_2}{\lambda m_1 + (1-\lambda)m_2 \kappa}\right)$$

which is non-negative, given Assumption 1. Hence, $\delta(\kappa) < 0$, $\kappa < 1$, which proves the result.

B Proof of Proposition 1

We divide the proof in several steps.

(i) Consider first that $\gamma > \hat{\gamma}^c(\kappa)$, that is, the bridge is not built in both regimes.

The welfare comparison boils down to

$$W^d(0) - W^c(0) = \lambda m_1 \ln\left(\frac{2\lambda m_1}{\lambda m_1 + (1-\lambda)m_2}\right) + \\ + (1-\lambda)m_2 \ln\left(\frac{2(1-\lambda)m_2}{\lambda m_1 + (1-\lambda)m_2}\right)$$

Let $\lambda m_1 = a$ and $(1-\lambda)m_2 = b \leq a$ and notice that, when $a = b$, $W^d(0) - W^c(0) = 0$. Furthermore,

$$\frac{\mathrm{d}\left(W^d(0) - W^c(0)\right)}{\mathrm{d}a} = \ln(2a) - \ln(a+b) > 0$$

Hence, $W^d(0) - W^c(0) \geq 0$.

(ii) Consider now that $\gamma < \hat{\gamma}^d(\kappa)$, so that both regimes build the bridge. The welfare difference is equal to $W^d(\kappa) - W^c(\kappa)$. From (i) we know that $W^d(0) - W^c(0) \geq 0$, and moreover

$$\begin{aligned}\frac{\mathrm{d}\left(W^d(\kappa) - W^c(\kappa)\right)}{\mathrm{d}\kappa} &= \frac{\mathrm{d}\left(W^d(\kappa) - W^o(\kappa)\right)}{\mathrm{d}\kappa} - \frac{\mathrm{d}\left(W^c(\kappa) - W^o(\kappa)\right)}{\mathrm{d}\kappa} \\ &= \frac{\mathrm{d}\left(\hat{\gamma}^d(\kappa) - \hat{\gamma}^o(\kappa)\right)}{\mathrm{d}\kappa} - \frac{\mathrm{d}\left(\hat{\gamma}^c(\kappa) - \hat{\gamma}^o(\kappa)\right)}{\mathrm{d}\kappa}\end{aligned}$$

which is strictly smaller than 0, as shown in Appendix A. Finally, after straightforward simplification, one obtains

$$W^d(1) - W^c(1) = [\lambda m_1 + (1-\lambda)m_2]\left[1 + \ln\left(\frac{\lambda(1-\lambda)m_1 m_2}{(\lambda m_1 + (1-\lambda)m_2)^2}\right)\right]$$

We now show that $(a+b)^2/ab > e$, which proves that $W^d(1) - W^c(1) < 0$. Observe that

$$\frac{(a+b)^2}{ab} = 2 + \frac{a^2+b^2}{ab} \geq 2 + \frac{2b^2}{b^2} = 4 > e$$

This shows that there is a threshold value of κ above which centralization dominates decentralization.

(iii) The final case is for intermediate values $\hat{\gamma}^d(\kappa) \leq \gamma \leq \hat{\gamma}^c(\kappa)$. In this case, the welfare difference is equal to $W^d(0) - W^c(\kappa) + \gamma$. Firstly, note that $W^d(0) -$

$W^c(0) + \gamma > 0$, using (i) above. Secondly,

$$\frac{\mathrm{d}\big(W^d(0) - W^c(\kappa) + \gamma\big)}{\mathrm{d}\kappa} = -\frac{\mathrm{d}W^c(\kappa)}{\mathrm{d}\kappa} < 0$$

Consider now $\kappa = 1$, and suppose that $\gamma = \hat{\gamma}^d(\kappa)$. Hence,

$$W^d(0) - W^c(1) + \gamma = W^d(0) - W^c(1) + W^d(1) - W^d(0) < 0$$

where the inequality follows from (ii). Analogously, for $\gamma = \hat{\gamma}^c(\kappa)$,

$$W^d(0) - W^c(1) + \gamma = W^d(0) - W^c(1) + W^c(1) - W^c(0) > 0$$

Hence, there is a threshold value $\tilde{\gamma} < \hat{\gamma}^c(\kappa)$ such that $W^d(0) - W^c(1) + \gamma > 0, \forall \gamma > \tilde{\gamma}$. We have that, for $\gamma > \tilde{\gamma}$, $W^d(0) - W^c(\kappa) + \gamma > 0, \forall \kappa$. For lower values of γ, there is a threshold value of κ above which centralization dominates. This threshold is increasing in γ.

C Proof of Lemma 2

It is straightforward to obtain

$$\hat{\gamma}^{pc1}(\kappa) - \hat{\gamma}^{pd1}(\kappa) = \kappa\, m_1 \left[\ln\left(\frac{\kappa\, m_1}{(1-\lambda)m_2} \right) - 1 \right]$$

which is greater than 0 if and only if $\kappa > e(1-\lambda)m_2/m_1$. Notice that this threshold is above 1 when $m_1/((1-\lambda)m_2) < e$, in which case $\hat{\gamma}^{pc1}(\kappa) - \hat{\gamma}^{pd1}(\kappa) \leq 0, \forall \kappa$.

D Proof of Lemma 3

It is straightforward to obtain

$$\hat{\gamma}^{pc2}(\kappa) - \hat{\gamma}^{pd2}(\kappa) = \kappa\, m_2 \left[\ln\left(\frac{\kappa\, m_2}{\lambda m_1} \right) - 1 \right]$$

which is greater than 0 if and only if $\kappa > e\lambda m_1/m_2$. Notice that this threshold is above 1 when $\lambda m_1/m_2 > e^{-1}$, in which case $\hat{\gamma}^{pc2}(\kappa) - \hat{\gamma}^{pd2}(\kappa) \leq 0$, $\forall \kappa$.

E Proof of Proposition 2

Let us consider first the case where district 1 is majoritarian.

(i) When both regimes build the bridge, total welfare (excluding the provision cost of the bridge) is given by

$$\begin{aligned} W^{pd1}(\kappa) = \lambda m_1 & \left[\ln\left(\frac{\lambda m_1}{\rho}\right) + \kappa \ln\left(\frac{(1-\lambda)m_2}{\rho}\right) \right] + \\ & + (1-\lambda)m_2 \left[\ln\left(\frac{(1-\lambda)m_2}{\rho}\right) + \kappa \ln\left(\frac{\lambda m_1}{\rho}\right) \right] - \\ & - [\lambda m_1 + (1-\lambda)m_2] \end{aligned}$$

and,

$$\begin{aligned} W^{pc1}(\kappa) = \lambda m_1 & \left[\ln\left(\frac{m_1}{\rho}\right) + \kappa \ln\left(\frac{m_1 \kappa}{\rho}\right) \right] + \\ & + (1-\lambda)m_2 \left[\ln\left(\frac{m_1 \kappa}{\rho}\right) + \kappa \ln\left(\frac{m_1}{\rho}\right) \right] - \\ & - (m_1 + m_1 \kappa) \end{aligned} \tag{14}$$

Firstly, note that $\lim_{\kappa \to 0} W^{pc1}(\kappa) = -\infty$. Hence $W^{pd1}(0) > W^{pc1}(0)$. Now,

$$\begin{aligned} \frac{\mathrm{d}\left(W^{pd1}(\kappa) - W^{pc1}(\kappa)\right)}{\mathrm{d}\kappa} &= \lambda m_1 \ln\left(\frac{(1-\lambda)m_2}{m_1 \kappa}\right) + \\ +(1-\lambda)m_2 \ln\lambda + \frac{1-\lambda}{\kappa}(m_1 \kappa - m_2) &= \delta(\kappa) \end{aligned}$$

Moreover,

$$\frac{\mathrm{d}\delta(\kappa)}{\mathrm{d}\kappa} = \kappa^{-2}[(1-\lambda)m_2 - \lambda m_1 \kappa]$$

which is positive (resp., negative) for $\kappa < \tilde{\kappa} = (1-\lambda)m_2/(\lambda m_1)$ (resp., $\kappa > \tilde{\kappa}$). Also, $\delta(\tilde{\kappa}) = [\lambda m_1 + (1-\lambda)m_2]\ln(\lambda) + m_1(1-2\lambda) < 0$, given $\lambda > 1/2$. Hence, $\delta(\kappa) < 0$, $\forall \kappa$.

Finally, let us show that $W^{pd1}(1) - W^{pc1}(1) < 0$.

$$W^{pd1}(1) - W^{pc1}(1) = [\lambda m_1 + (1-\lambda)m_2] \ln\left(\frac{\lambda(1-\lambda)m_2}{m_1}\right) + \\ + (2-\lambda)m_1 - (1-\lambda)m_2 \tag{15}$$

Straightforward computations allow one to show that (15) is decreasing in λ. When $\lambda = 1/2$, (15) boils down to

$$\frac{1}{2}\left[(m_1 + m_2)\ln\left(\frac{m_2}{4m_1}\right) + 3m_1 - m_2\right] \tag{16}$$

which is a decreasing function of m_1. Setting m_1 to its lowest possible value, $m_1 = m_2$, (16) is equal to $m_2[1 - \ln(4)] < 0$. Hence, (15) is always negative, and there exists a value of κ above which centralization dominates decentralization.

(ii) Whenever the centralized regime does not build the bridge, $W^{pc1}(0) \to -\infty$, hence decentralization dominates.

We now address the case when district 2 is majoritarian.

(i) When both regimes build the bridge, total welfare (excluding the provision cost of the bridge) is given by

$$W^{pd2}(\kappa) = \lambda m_1\left[\ln\left(\frac{\lambda m_1}{\rho}\right) + \kappa \ln\left(\frac{(1-\lambda)m_2}{\rho}\right)\right] + \\ + (1-\lambda)m_2\left[\ln\left(\frac{(1-\lambda)m_2}{\rho}\right) + \kappa \ln\left(\frac{\lambda m_1}{\rho}\right)\right] - \\ - [\lambda m_1 + (1-\lambda)m_2]$$

and,

$$W^{pc2}(\kappa) = \lambda m_1\left[\ln\left(\frac{m_2\,\kappa}{\rho}\right) + \kappa \ln\left(\frac{m_2}{\rho}\right)\right] + \\ + (1-\lambda)m_2\left[\ln\left(\frac{m_2}{\rho}\right) + \kappa \ln\left(\frac{m_2\,\kappa}{\rho}\right)\right] - \\ - (m_2 + m_2\,\kappa)$$

Firstly, notice that $\lim_{\kappa\to 0} W^{pc2}(\kappa) = -\infty$ and hence $W^{pd2}(0) > W^{pc2}(0)$. Now,

$$\frac{\mathrm{d}\left(W^{pd2}(\kappa) - W^{pc2}(\kappa)\right)}{\mathrm{d}\kappa} = \lambda m_1 \ln(1-\lambda) + \\ +(1-\lambda) m_2 \ln\left(\frac{\lambda m_1}{m_2\,\kappa}\right) - \lambda\left(\frac{m_1}{\kappa} - m_2\right) = \delta(\kappa)$$

Moreover,

$$\frac{\mathrm{d}\delta(\kappa)}{\mathrm{d}\kappa} = \kappa^{-2}[\lambda m_1 - (1-\lambda) m_2 \kappa]$$

which is positive (resp., negative) for $\kappa < \tilde{\kappa} = \lambda m_1/[(1-\lambda)m_2]$ (resp., $\kappa > \tilde{\kappa}$). Also, $\delta(\tilde{\kappa}) = [\lambda m_1 + (1-\lambda)m_2]\ln(1-\lambda) - m_2(1-2\lambda) < 0$. Hence, $\delta(\kappa) < 0$, $\forall \kappa$.

Let $\overline{\kappa} = \lambda m_1/m_2 < \tilde{\kappa}$. We now show that $W^{pd2}(\overline{\kappa}) - W^{pc2}(\overline{\kappa}) < 0$, which implies that there exists a critical value of κ between 0 and $\overline{\kappa} < \tilde{\kappa}$ above which centralization dominates decentralization. After straightforward simplification, one obtains

$$W^{pd2}(\overline{\kappa}) - W^{pc2}(\overline{\kappa}) = \frac{\lambda^2 m_1^2 + (1-\lambda) m_2^2}{m_2} \ln(1-\lambda) + \lambda m_2$$

which is a decreasing function of m_1. Setting m_1 to its lowest value, $m_1 = m_2$, the expression boils down to $m_2[(\lambda^2 + 1 - \lambda)\cdot \ln(1-\lambda) + \lambda]$ which is negative for $\lambda < 1/2$. This shows that $W^{pd2}(\overline{\kappa}) - W^{pc2}(\overline{\kappa}) < 0$, $\forall m_1 > m_2$.

(ii) Whenever the centralized regime does not build the bridge, $W^{pc2}(\kappa) \to -\infty$, hence decentralization dominates.

F Proof of Proposition 3

Let us consider first that district 1 is majoritarian. Then, the case where the centralized system builds the bridge, while the decentralized one does not, may only happen for $\kappa > e(1-\lambda)m_2/m_1$, and $\gamma \in (\hat{\gamma}^{pd1}(\kappa), \hat{\gamma}^{pc1}(\kappa))$. Firstly, notice that welfare with decentralization is given by

$$W^{pd1}(0) = \lambda m_1 \ln\left(\frac{\lambda m_1}{\rho}\right) + (1-\lambda) m_2 \ln\left(\frac{(1-\lambda) m_2}{\rho}\right) - [\lambda m_1 + (1-\lambda) m_2]$$

whereas welfare with centralization is given by (14). Hence, $W^{pd1}(0) > W^{pc1}(\kappa) - \gamma$ if and only if $\gamma > \tilde{\gamma}$, where $\tilde{\gamma}$ is defined as

$$\tilde{\gamma} = -\lambda m_1 \ln\lambda + (1-\lambda)m_2 \ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + \lambda m_1 + (1-\lambda)m_2 + \\ + \lambda m_1\,\kappa \ln\left(\frac{m_1\,\kappa}{\rho}\right) + (1-\lambda)m_2\,\kappa\ln\left(\frac{m_1}{\rho}\right) - m_1 - m_1\,\kappa$$

Note that we must have $\tilde{\gamma} \in \left(\hat{\gamma}^{pd1}, \hat{\gamma}^{pc1}\right)$. Now

$$\begin{aligned}\tilde{\gamma} - \hat{\gamma}^{pc1} &= -\lambda m_1 \ln\lambda + (1-\lambda)(m_2 - m_1) + \\ &+ (1-\lambda)m_2 \ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + (1-\lambda)m_2\,\kappa \ln m_1 - \\ &- (1-\lambda)m_1\,\kappa\ln(m_1\,\kappa) + \kappa(1-\lambda)(m_1 - m_2)\ln\rho\end{aligned} \tag{17}$$

and,

$$\begin{aligned}\tilde{\gamma} - \hat{\gamma}^{pd1} &= -\lambda m_1 \ln\lambda + (1-\lambda)(m_2 - m_1) + \\ &+ (1-\lambda)m_2 \ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + (1-\lambda)m_2\,\kappa \ln m_1 + \\ &+ \lambda m_1\,\kappa\ln(m_1\,\kappa) - m_1\,\kappa \ln\big((1-\lambda)m_2\big) - m_1\,\kappa + \\ &+ \kappa(1-\lambda)(m_1 - m_2)\ln\rho\end{aligned} \tag{18}$$

from which it is obvious that $\exists\,\hat{\rho}^{pc1}$ such that $\tilde{\gamma} < \hat{\gamma}^{pc1}$, $\forall \rho < \hat{\rho}^{pc1}$ and $\exists\,\hat{\rho}^{pd1}$ such that $\tilde{\gamma} > \hat{\gamma}^{pd1}$, $\forall \rho > \hat{\rho}^{pd1}$.

We need to make sure that *(a)* $\hat{\rho}^{pd1} < \hat{\rho}^{pc1}$, *(b)* $\hat{\rho}^{pd1} > 0$, and $\hat{\rho}^{pc1} > 0$, and finally, *(c)* $\hat{\rho}^{pd1} < (1-\lambda)m_2$, and $\hat{\rho}^{pc1} < (1-\lambda)m_2$. We tackle each of these in turn:

(a) Follows straightforwardly from the fact that the difference between the right-hand sides of (17) and (18) is

$$m_1\,\kappa\left[\ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) - 1\right] = \hat{\gamma}^{pc1} - \hat{\gamma}^{pd1} > 0$$

(b) Notice that when $\rho = 0$, $\tilde{\gamma} - \hat{\gamma}^{pc1} \to -\infty$ and $\tilde{\gamma} - \hat{\gamma}^{pd1} \to -\infty$, which ensures that $\hat{\rho}^{pc1} > 0$ and $\hat{\rho}^{pd1} > 0$.

(c) We now show that $\hat{\rho}^{pc1} < (1-\lambda)m_2$, and $\hat{\rho}^{pd1} < (1-\lambda)m_2$ for sufficiently high m_2. We begin by looking at the difference between $\tilde{\gamma}$ and $\hat{\gamma}^{pc1}$ when $\rho = (1-\lambda)m_2$,

$$\begin{aligned}&(1-\lambda)(m_2 - m_1) + (1-\lambda)(m_2 - m_1\,\kappa)\ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + \\ &+(1-\lambda)m_2\,\kappa\ln\left(\frac{m_1}{(1-\lambda)m_2}\right) - \lambda m_1 \ln\lambda\end{aligned} \tag{19}$$

which, deriving with respect to m_2 is equal to

$$(1-\lambda)\left[\ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + \kappa\ln\left(\frac{m_1}{(1-\lambda)m_2 e}\right) + \frac{m_1\,\kappa}{m_2}\right] > 0$$

where we have used the fact that $\kappa > (1-\lambda)m_2 e/m_1 \Rightarrow m_1 > (1-\lambda)m_2 e$. Letting m_2 vary between 0 and m_1, (19) varies between $-\infty$ and

$$-\lambda m_1 \ln\lambda + (1-\lambda)(1-\kappa)m_1\ln\left(\frac{\kappa}{1-\lambda}\right) - (1-\lambda)m_1\,\kappa\ln(1-\lambda) > 0$$

We now look at the difference between $\tilde{\gamma}$ and $\hat{\gamma}^{pd1}$ when $\rho = (1-\lambda)m_2$,

$$\begin{aligned}&(1-\lambda)(m_2 - m_1) + [(1-\lambda)m_2 + \lambda m_1\,\kappa]\ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + \\ &+(1-\lambda)m_2\,\kappa\ln\left(\frac{m_1}{(1-\lambda)m_2}\right) - m_1\,\kappa - \lambda m_1\ln\lambda\end{aligned} \tag{20}$$

which, deriving with respect to m_2 is equal to

$$(1-\lambda)\left[\ln\left(\frac{m_1\,\kappa}{(1-\lambda)m_2}\right) + (1-\lambda)\kappa\ln\left(\frac{m_1}{(1-\lambda)m_2 e}\right)\right] + \frac{\lambda m_1\,\kappa}{m_2} > 0$$

where we have used the fact that $\kappa > (1-\lambda)m_2 e/m_1 \Rightarrow m_1 > (1-\lambda)m_2 e$. Letting m_2 vary between 0 and m_1, (20) varies between $-\infty$ and

$$\begin{aligned}&-\lambda m_1\ln\lambda + (1-\lambda)m_1\ln\left(\frac{\kappa}{1-\lambda}\right) - (1-\lambda)m_1\,\kappa\ln(1-\lambda) + \\ &+ m_1\,\kappa\left[\lambda\ln\left(\frac{\kappa}{1-\lambda}\right) - 1\right] > 0\end{aligned}$$

where we have used the fact that $\kappa > (1-\lambda)m_2 e/m_1 \Rightarrow \kappa > (1-\lambda)e$ when $m_1 = m_2$.

Let us consider now that district 2 is majoritarian. In this situation, $\kappa > e\lambda m_1/m_2 = \tilde{\kappa}$, and $\gamma \in (\hat{\gamma}^{pd2}(\kappa), \hat{\gamma}^{pc2}(\kappa))$. Notice that $\tilde{\kappa} < 1$ if and only if $m_1 < \overline{m_1} = m_2/(e\lambda)$.

Firstly, notice that welfare levels (excluding the provision cost of the bridge, if built) in both regimes are given by

$$\begin{aligned} W^{pd2}(0) &= \lambda m_1 \ln\left(\frac{\lambda m_1}{\rho}\right) + (1-\lambda)m_2 \ln\left(\frac{(1-\lambda)m_2}{\rho}\right) - [\lambda m_1 + (1-\lambda)m_2] \\ W^{pc2}(\kappa) &= \lambda m_1 \left[\ln\left(\frac{m_2\,\kappa}{\rho}\right) + \kappa \ln\left(\frac{m_2}{\rho}\right)\right] + \\ &\quad + (1-\lambda)m_2 \left[\ln\left(\frac{m_2}{\rho}\right) + \kappa \ln\left(\frac{m_2\,\kappa}{\rho}\right)\right] - (m_2 + m_2\,\kappa) \end{aligned}$$

Hence, $W^{pd2}(0) > W^{pc2}(\kappa) - \gamma$ if and only if $\gamma > \tilde{\gamma}$, with $\tilde{\gamma}$ defined as

$$\begin{aligned} \tilde{\gamma} = &-(1-\lambda)m_2 \ln(1-\lambda) - \lambda m_1 \ln\left(\frac{\lambda m_1}{m_2\,\kappa}\right) + \lambda(m_1 - m_2) + \\ &+ \lambda m_1\,\kappa \ln\left(\frac{m_2}{\rho}\right) + (1-\lambda)m_2\,\kappa \ln\left(\frac{m_2\,\kappa}{\rho}\right) - m_2\,\kappa \end{aligned}$$

We now show that $\tilde{\gamma} > \hat{\gamma}^{pc2}(\kappa)$, which implies that $W^{pd2}(0) < W^{pc2}(\kappa) - \gamma$ under all relevant parameter values. Straightforward simplification allows one to write $\tilde{\gamma} - \hat{\gamma}^{pc2}(\kappa)$ as

$$\begin{aligned} &-(1-\lambda)m_2 \ln(1-\lambda) - \lambda m_1 \ln\left(\frac{\lambda m_1}{m_2\,\kappa}\right) + \lambda(m_1 - m_2) + \\ &+ \lambda m_1\,\kappa \ln\left(\frac{m_2}{\rho}\right) - \lambda m_2\,\kappa \ln\left(\frac{m_2\,\kappa}{\rho}\right) \end{aligned} \tag{21}$$

which is an increasing function of m_1, given $\kappa > e\lambda m_1/m_2$. Setting $m_1 = m_2$, the expression boils down to

$$-(1-\lambda)m_2 \ln(1-\lambda) + \lambda m_2 \ln(\kappa/\lambda) - \lambda m_2\,\kappa \ln(\kappa)$$

which is always positive for the relevant values of κ and for $\lambda < 1/2$.